The Universe without Dark Matter

Abstract

The currently accepted model which describes the evolution of the universe (the ΛCDM model) assumes the existence of what is called Cold Dark Matter. The main problem is that nobody has a clue what this matter looks like and no one has ever detected any. This article explores the implications of a universe without matter. Alternative explanations are suggested for the anomalous rotation curves of the galaxies, gravitational lensing and the apparent flatness of the universe.

The age of the universe

Using the ΛCDM model cosmologists are confident that the Big Bang occurred 13.8 billion years ago plus or minus about 2%. This is an amazingly accurate result considering the immensely complicated calculations which underpin it. The most fundamental observation contributing to this conclusion is the fact that distant galaxies show a consistent red shift in their spectra and the magnitude of this shift appears to be proportional to their distance from us. Initially the red shift was interpreted as the Doppler shift caused by a recession velocity. By dividing the velocity by the distance it is possible to calculate the constant of proportionality H_0 . The value of this constant as measured by direct observation of individual galaxies lies between 73 and 76 km/s/Mpc. Measurements based on the Cosmic Microwave Background radiation (CMB) lead to a slightly smaller value of 67 to 68 km/s/Mpc. Naturally astronomers favour the former value while cosmologists tend to prefer the latter! The best we can say is that the true value is probably about 72 km/s/Mpc plus or minus 6%.

Since 1 Mpc = 3.1×10^{19} km, this translates into 2.3×10^{-18} s^{-1}. The reciprocal of this has the dimensions of time and is equal to 4.3×10^{17} seconds or 13.6 billion years plus or minus a billion years. This can be interpreted as the time it would have taken the universe to expand assuming that its rate of expansion was constant. As we shall see, the rate of expansion has not been constant but by a remarkable coincidence, this figure is very close to the age of the universe as predicted by our best current models.

It is now recognised that the red shift is not due to the galaxies actually moving through a static space but that space itself is expanding. This does not materially alter the conclusion, however, that the universe was once very, possibly infinitely, dense.

The CMB

The most important piece of observational evidence relevant to the history of the universe is undoubtedly the detailed study of the Cosmic Microwave Background radiation. This was discovered by Penzias and Wilson in 1964 and consists of an almost completely uniform glow of radiation over the whole sky whose spectrum corresponds very closely to that of a black body at a temperature of 2.7 K. This is interpreted as photons released when the universe cooled to the point at which electrons and protons could recombine to form neutral atoms. This happens at a temperature of 3000 K. Since the wavelength of a photon is inversely proportional to its energy and temperature, this means that the photons have been stretched by a factor of 3000/2.7 = 1100 since that time.

The Friedman equation

Einstein's theory of General Relativity predicts how a universe filled with energy and radiation will evolve. Basically, matter and energy cause space to curve in on itself resulting in a universe in which the angles of a triangle would add up to more than 180° (like the angles of a triangle on the surface of a sphere). Matter also causes any expansion to slow down. All the evidence we have, however, points to the fact that our universe is actually spatially flat (I shall discuss the evidence for this later) and that, if anything, the rate of expansion is speeding up, not slowing down. This implies the existence of a factor (which is called dark energy) which has the opposite effect to matter. This results in the following equation:

$$H^2 = \frac{1}{3}\Lambda + \frac{8\pi G}{3}\rho_m \qquad (1)$$

H is the Hubble parameter (we cannot call it a constant because we are allowing the rate of expansion of the universe to change with time); Λ (lambda) is the constant which represents dark energy (this is genuinely constant according to GR); G is the Newtonian constant of gravitation and ρ_m is the density of matter (and energy) in the universe at any point in time.

Now since the amount of matter (and energy) is assumed to remain constant over time, as the universe expands, the density of matter, ρ_m, will decrease. This means that in any universe containing matter, the Hubble parameter will get smaller and smaller. The universe will still go on expanding, but more and more slowly.

It is possible to rearrange this equation to put the two terms into the same form:

$$H^2 = \frac{8\pi G}{3}\rho_\Lambda + \frac{8\pi G}{3}\rho_m = \frac{8\pi G}{3}(\rho_\Lambda + \rho_m)$$
$$(\rho_\Lambda + \rho_m) = \frac{3H^2}{8\pi G} \qquad (2)$$

It is convenient to define a quantity quantity $\rho_{crit} = \frac{3H_0^2}{8\pi G}$ called the *critical density* of the universe where H_o is the value of the Hubble parameter at the present time. We can calculate the value of ρ_{crit} by putting H_0 equal to 2.3×10^{-18} s^{-1} and $G = 6.67 \times 10^{-11}$ m^3/kg. It works out to be 9.5×10^{-27} kg/m^3. Since a proton has a mass of 1.6×10^{-27} kg, this means that a universe which consisted of matter (and had a cosmological constant of zero) would need about 6 hydrogen atoms per cubic metre in order to match its critical density and render it spatially flat.

Now astronomers have estimated the density of matter in our own universe and have found it difficult to make it any greater than 5% of the critical density. This is the main reason why we are forced to concede that the cosmological constant is not zero. For reasons which we shall come to later, cosmologists think there may be more matter in the universe (called dark matter) than the matter that astronomers can see, but not enough to enable us to abandon the cosmological constant.

Using ρ_{crit} we can make one further simplification to equation (2). Let us define $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{crit}}$ and $\Omega_m = \frac{\rho_m}{\rho_{crit}}$. We now have simply $\Omega_\Lambda + \Omega_m = (H/H_0)^2$ or:

$$H^2 = H_0^2(\Omega_\Lambda + \Omega_m) \qquad (3)$$

Note that while Ω_Λ is strictly constant, Ω_m decreases as the universe expands according to an inverse cube law.

We are now in a position to calculate the age of the universe by extrapolation equation (3) backwards in time. We need to introduce an arbitrary scale factor a whose value is 1 at the current time and (obviously) 0 at the Big Bang. (a may be regarded as a standard length.) The Hubble parameter is a measure of the rate of change of a (i.e. da/dt) but in order to give it the dimensions of s^{-1}, we must divide by a so $H = \frac{da/dt}{a}$. The Λ term is constant but we must replace Ω_m with $\frac{\Omega_{m0}}{a^3}$ where Ω_{m0} is the current proportion of matter (including dark matter) density to the critical density. Equation (3) now becomes:

$$H^2 = \left(\frac{da/dt}{a}\right)^2 = H_0^2(\Omega_\Lambda + \frac{\Omega_{m0}}{a^3}) \quad (4)$$

This can be further simplified to:

$$\frac{da}{dt} = a H_0 \sqrt{\Omega_\Lambda + \frac{\Omega_{m0}}{a^3}} \quad (5)$$

where $\Omega_\Lambda + \Omega_{m0} = 1$

In a universe in which there is no matter (i.e. the critical density is due entirely to the cosmological constant) the we have $\Omega_{m0} = 0$ and $\Omega_\Lambda = 1$:

$$\frac{da}{dt} = a H_0 \quad (6)$$

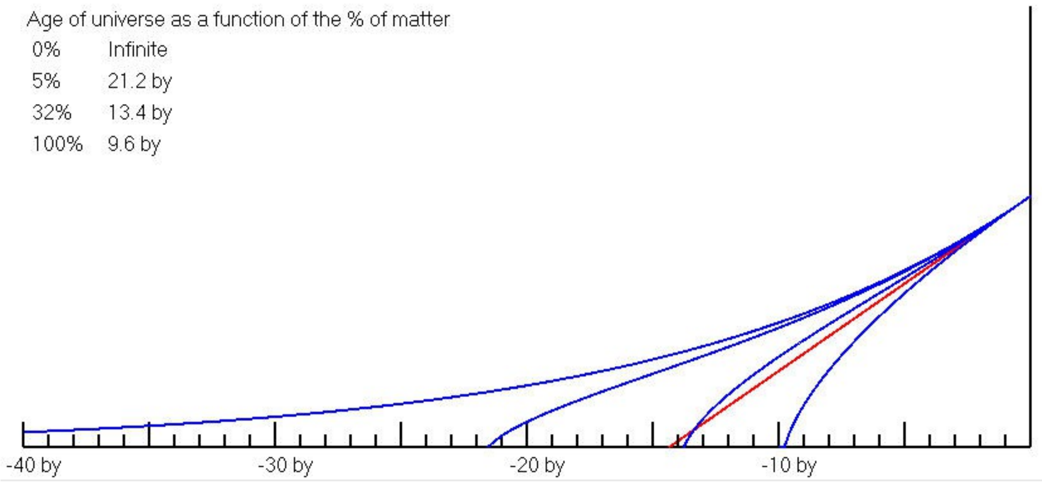
When the rate of change of a quantity is proportional to the quantity itself the result is an exponential rise. Such a universe would be infinitely old.

In a universe which the cosmological constant is zero and which contains the critical density of matter, $\Omega_{m0} = 1$ and

$$\frac{da}{dt} = H_0 a^{-1/2} \quad (7)$$

This results in a universe which obeys a 2/3 power law and which is about 9.6 billion years old. This is far too short for the astronomers who think that many globular clusters are much older than this.

By adjusting the value of Ω_{m0} the following expansion curves are obtained. Of particular interest are the case when Ω_{m0} = 32% (which is the currently accepted value) and Ω_{m0} = 5% which represents a universe with no dark matter.



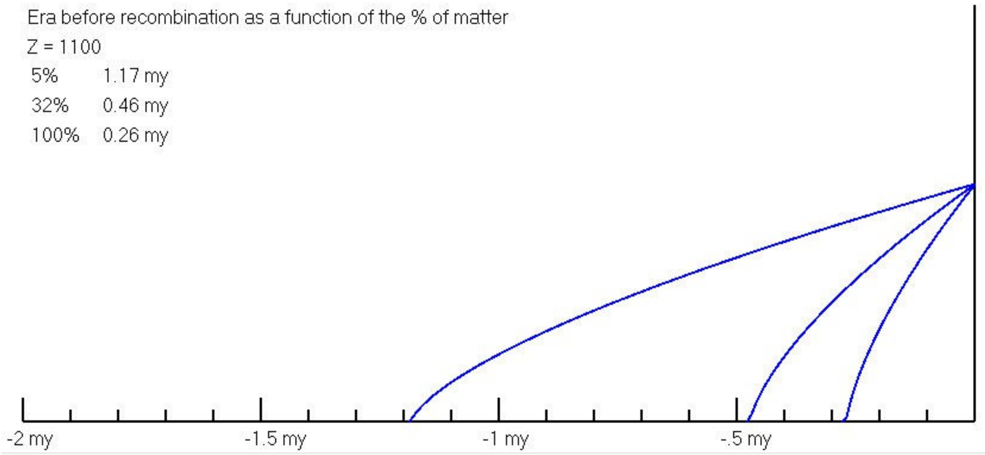
(The red line extrapolates back to a Hubble time of 14.4 billion years.)

As I remarked before, the currently preferred scenario puts the age of the universe almost exactly at the Hubble Time of around 14 billion years.

A (flat) universe which has no dark matter would be much older - about 21 billion years but, as we shall see, I have reason to believe that the universe is not flat.

The era before recombination

One figure that is of great importance is the time of the recombination event. We know that this happened when *a* had the value of 1/1100 = 0.00091. If the universe had expanded linearly (i.e. along the red line) then its age at recombination would have been 14.4 billion / 1100 = 13 million years after the Big Bang. But you will note that the blue curves meet the axis at a steeper angle meaning that in the initial phase the universe is expanding more rapidly. The reason for this is that in the early universe the density of matter was much greater than it is now[1]. This reduces the time of the recombination event. The following graphs show the expansion of the universe during the recombination era.



In a universe with no dark energy (i.e. 100% matter) the recombination event would have occurred 270,000 years after the big bang. Using the currently accepted value of *s* (32%) we obtain a figure of 460,000 years. This is rather larger than the often quoted figure of 380,000 years. The discrepancy is due to the fact that during the early universe there was considerable extra pressure due to radiation which would cause the universe to expand more rapidly than the simple model employed here. In a universe with no dark matter, the recombination event would have occurred around a million years after the Big Bang.

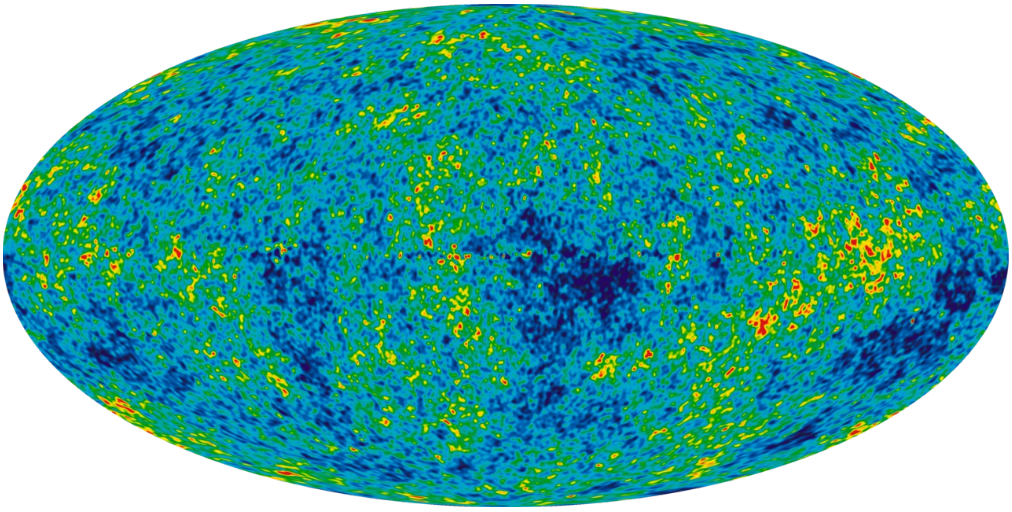
Is the universe flat?

In all of the previous discussion we have been assuming that the universe is spatially flat. Originally it was thought that the universe contained only matter (or rather mass/energy) and since there appears to be far less of this than is required to make the universe flat, the universe was thought to be negatively curved. Cosmologists now believe it to be as near to being flat as we can detect, the missing matter being replaced partly by dark matter and partly by a non-zero cosmological constant. What then is the evidence that the universe is flat?

One way to find out would be to survey the angles of a huge triangle using trig points on a couple of nearby galaxies. Obviously we do not have the technology to do this yet but there is another way.

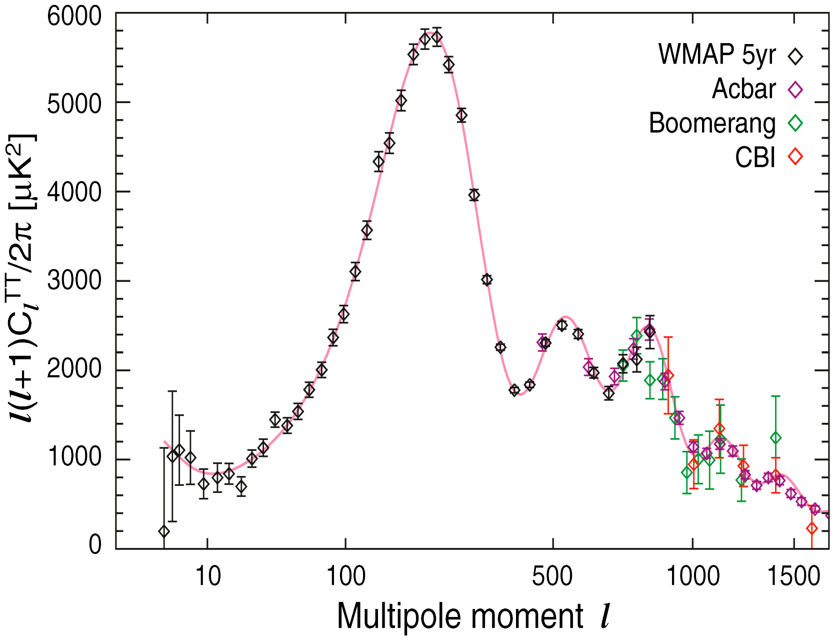
When the CMB was mapped with great precision and in great detail by instruments such as the Wilkinson Anisotropy Probe (WMAP) it was discovered that the temperature was not exactly uniform but showed blotch variations of a few millionths of a degree Kelvin as shown in this iconic image:

1 Note that this faster expansion has nothing to do with the extremely rapid expansion known as inflation which may (or may not) have occurred during the first few microseconds after the Big Bang.



The image can be thought of as a map of the surface of the universe as it was at the time of recombination with the slightly hotter (red) areas representing regions where matter is clumping together under gravity and the cooler (blue) areas representing regions of below average density.

There is obviously a huge amount of data hidden in this image but a lot of it is random. The best way to analyse it is by using a technique analogous to Fourier analysis which produces what is known as the power spectrum of the CMB. Here it is:



The most dominant feature of the map is its 'blobby nature'. Most of these 'blobs' are about 1 degree in diameter and it is these 'blobs' which are responsible for the large peak at an multipole moment of about $l = 220$. This corresponds roughly to an angular size of about 1.2°. The vertical axis is measured in μK and is a measure of the amplitude of the blobs at that size. The red line is a theoretical prediction based on various assumptions about the universe – principally that the universe is flat and that dark matter contributes some 27% of the critical density, the rest being provided by dark energy. The agreement between theory and practice is remarkable but it must be born in mind that the parameters of the ΛCDM ('lambda-cold-dark-matter') model have been specifically tweaked to produce the desired fit. It is not the case that the theoreticians predicted the spectrum in advance and it is possible that other models might fit the data just as well.

Evidence for the flatness of the universe comes from the existence and position of the first peak. Very roughly, since the universe has expanded by a factor of 1100 since the recombination event, the largest oscillations which the universe could accommodate at that time would now have an angular size in the sky of about 360/1100 = 0.3°. For reasons to do with the behaviour of acoustic waves in a hot plasma and also the spherical nature of these waves, mathematicians confidently predict that the angular size of the first peak would be exactly where it is – at 1.2°. They infer from this that the universe is flat because if the universe was curved, the blobs would be either magnified (in he case of positive curvature) or diminished. Since the data agrees precisely with the theoretical prediction, the conclusion is that the universe is flat. In short it is argued that the ΛCDM model (which takes as one of its basic assumptions that the universe contains the critical density of matter/dark energy and is therefore flat) predicts a peak in the power spectrum of 1.2°. Observations show that this peak exists. *Ergo* the universe is flat. The fallacy in this argument is obvious. You cannot use a model which assumes the very thing you are trying to prove. A very similar argument was employed in the eighteenth century when it was argued that the phlogiston theory (which assumes that phlogiston has negative weight) predicts that metals should gain weight when they are burned. Careful observations showed that this was indeed the case. *Ergo* phlogiston exists and has negative weight.

All I am trying to point out is that if you were to reject the idea that the universe is flat and adopt the heretical idea that dark matter does not exist, then all the arguments that predict a peak at l = 220 would have to be scrapped and worked out again from scratch.

Evidence for dark matter

The power spectrum also exhibits further peaks at l = 600 and l = 800. These are interpreted as 'harmonics' of the acoustic oscillations which existed at the time of the recombination event. Detailed calculations lead to the conclusion that the strength of these peaks is determined by the ratio of dark matter to baryonic matter. It is believed that a universe without dark matter would have a more prominent fundamental and very small harmonics peaks. Since the latter are quite significant, it is inferred that dark matter exists and it is these calculations which are largely responsible for the belief that the universe contains 5% baryonic matter, 27% dark matter and 68% dark energy.

Once again we must pour a little cold water on this argument. The figures quoted above have been chosen specifically to make the red curve 'fit the facts'. But this does not prove that the figures are correct any more than the success of the Ptolemaic theory of the Solar System proved that the Sun went round the Earth. In addition, by the time we reach the crucial third peak, the error bars are getting rather large so even the 'facts' are uncertain to a degree. In any case, although the *frequencies* of the harmonics of a physical system such as a drum skin are fixed by the physical size, shape and material of the skin, the relative *amplitudes* of the harmonics are affected by where, with what and how hard the drum is struck. I do not see how it is possible, even with a detailed knowledge of the physical structure of the early universe, to calculate the *amplitudes* of the various harmonics.

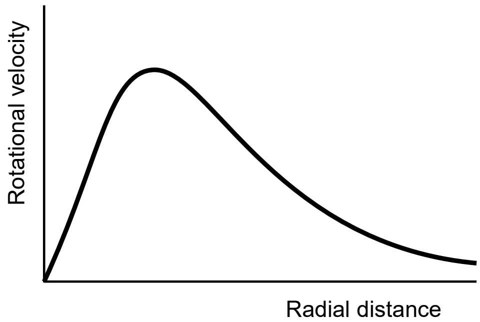
In short, the best we can say about the evidence from the CMB power spectrum is that the data are consistent with the parameters of the ΛCDM model but they do not *prove* either that the universe is flat or that dark matter exists.

Galactic Rotation Curves

More direct evidence for the existence of dark matter comes from the study of galactic rotation curves.

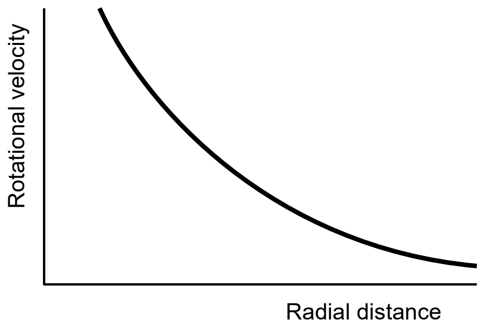
Over half of all the galaxies which we see in the sky are spiral galaxies. Many of them – possibly even the majority – have a supermassive Black Hole at their centre around which a multitude of stars make a stately rotation. Our own Sun is estimated to make a single revolution about the centre of our galaxy in about 250 million years. Since the Sun is about 26,000 light years from the centre this implies that the Sun is moving through space at a speed of around 200 km per hour or 0.065% of the speed of light.

In the 1960's and 70's it became possible to measure the rotation speed of individual stars in nearby galaxies (including our own) using the Doppler effect. When seen edge on, stars on one side of a galaxy will be moving towards us and the light from these stars will be blue-shifted. Stars on the opposite side of the galaxy will emit light which is red-shifted. After suitable corrections are made to account for the tilt of the galaxy, a graph may be plotted of the rotational speed of the stars against their distance from the centre. If we assume that gravity obeys an inverse square law we can make a prediction of the shape of this curve. Close to the galactic centre the rotational velocity of the stars will be very small because there is a relatively small amount of matter 'inside' the stars orbit. As we move away from the centre the rotational speed will rise because more and more matter will lie inside the orbit. At some point, however, the density of stars presumably begins to fall off and in consequence the rotational speed should begin to fall away and stars at a great distance from the galactic centre will, like the outer planets of our own solar system, rotate more and more slowly. The precise shape of the expected curve will depend on the precise way in which the density of the galaxy depends on the radial distance but the general shape will be something like this:



Galactric rotation curve for a galaxy without a Black Hole

We now know that many galaxies including our own have a supermassive Black Hole at their centre. For such a galaxy the rotation curve looks a little different. Instead of moving very slowly, the stars close to the Black Hole will have to rotate very rapidly. But stars a long way from the centre will not behave any differently from those in a galaxy without a Black Hole. Once the density of stars begins to fall away, the rotational speeds of the stars will fall away too. The galactic rotation curve will look something like this:



Galactic rotation curve for a galaxy with a Black Hole

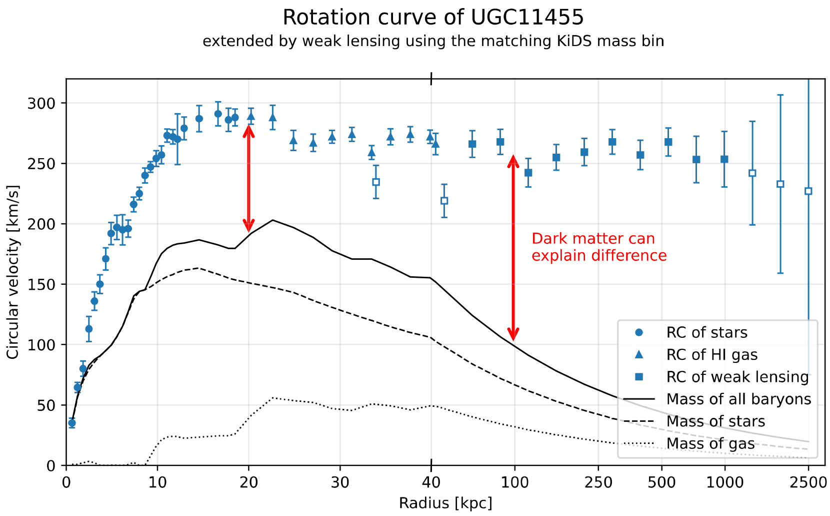
If we make the simplification that most of the mass of the galaxy lies in the central core (whether or not it contains a Black Hole) we can easily predict the shape of this curve. At any radius r the local gravitational field strength must be equal to the centripetal acceleration of the star; hence:

$$\frac{GM}{r^2} = \frac{v^2}{r}$$
$$v = \sqrt{\frac{GM}{r}} \quad (8)$$

In simple terms, a star twice as far from the galactic centre ought to rotate with a speed which is √2 times more slowly.

But in 1975 Vera Rubin, an astronomer at the Carnegie Institution of Washington, made an astonishing announcement. The outer stars of a large number of spiral galaxies all rotate *at the same speed*. Instead of falling off according to an inverse power law, the galactic rotation curves of these galaxies were basically flat.

Here is a modern rotation curve for the galaxy known as UGC11455 using the most up-to-date methods of measurement:



By ScienceDawns - Own work. Data from.[1][2], CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=180471173

There is clearly no falling off at all.

The traditional explanation for the discrepancy is that the galaxy must contain large amounts of extra matter which we cannot see – dark matter.

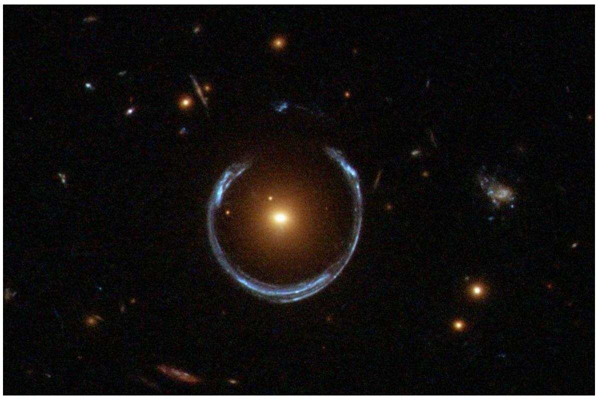
The question now arises – what distribution of matter would result in a flat rotation curve?

There are two cases to consider. In the first case we might suppose that the extra matter is concentrated in the plane of the galactic disc while the second possibility is that the dark matter is arranged in a halo of spherical shells around the galactic centre. The first case is very difficult to implement because the gravitational effect of a ring of material tends to infinity near the ring and its gravitational effects are not zero inside the ring. Spherical shells are, however, very easy to manipulate mathematically because the gravitational effect outside the shell is the same as that of a point source of gravity of the same mass while the gravitational effects inside the shell are everywhere zero. You can therefore just add more and more onion skins to the model to match any rotation curve you like.

The problem with this model is how to explain the stability of these spherical shells. Normally a gravitating object such as a planet, a star or a super-massive black hole surrounded by a cloud of gas will develop a disc of material round it in a plane which is at right angles to the total angular momentum of the cloud. This is because material orbiting within the disc can move without colliding with other material within the disc. Any object whose orbit is not in the plane of the disc must of necessity pass through the disc twice in every revolution. If the object collides with an object in the disc it will change its orbit. The only stable orbits are those within the disc. This is why Saturn's rings are so thin. That is the only way the particles within the ring can avoid colliding with each other. If these halos of dark matter exist, there must be a reason why the particles of dark matter do not collide with material in the disc. One possibility is that they simply do not interact with matter and, like neutrinos, pass through the disc without being affected in any way. Particles with this property are called WIMPs or Weakly Interacting Massive Particles.

Gravitational lensing

A third strand of evidence for the existence of dark matter comes from the phenomenon of gravitational lensing. When light passes close to a massive object the gravitational distortion of space and time generated by the object causes the light to bend. Famously, the bending of starlight passing close to the Sun was used to verify Einstein's General Theory back in the 1920's. Since then, many examples of light from a distant galaxy which happens to lie right behind a galaxy in the foreground have been discovered and it is easy to calculate the amount of matter needed in the foreground galaxy to cause the observed degree of bending. Typically, the calculations reveal that there is far more matter in the foreground galaxy than we can actually see.



An example of gravitational lensing photographed by the HST in 2007

Taken together, these three independent pieces of evidence – the flatness of the universe, galactic rotation and gravitational lensing – provide powerful evidence that dark matter exists. The trouble is – we can't find it! If it is in the form of some sort of exotic baryonic matter, it ought to tun up in our particle accelerators; primordial black holes ought to reveal their presence by exploding from time to time; WIMPs would be very difficult if not impossible to detect but trying to explain the behaviour of the universe by invoking the existence of particles which are impossible to detect is little short of mysticism.

I cannot help feeling that we are barking up the wrong tree though. The ΛCDM model fits the perceived facts so well that cosmologists have essentially stopped looking for alternative models. There have been attempts to modify Einstein's theory of gravity but the latter theory is so elegant and so successful it seems almost sacrilegious to tinker with it. Instead I wish to explore another line of enquiry, one which not only denies the flatness of the universe but also denies the assumption that it is homogeneous (i.e. exactly the same everywhere). Before you throw this essay into the bin, give me a chance to explain. We shall start by describing an alternative way in which the anomalous rotation of galaxies might be explained without the use of dark matter.

Anomalous galaxies

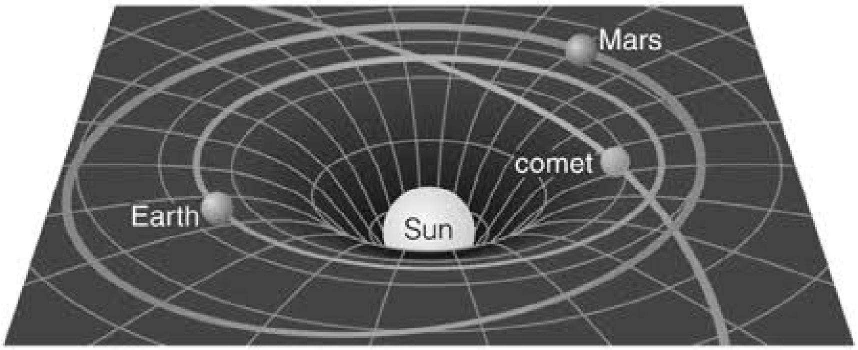
What exactly can we infer when we see the outer stars rotating more quickly than we expect? Does this *prove* that Dark Matter exists inside the galaxy? No, it does not. All that it shows is that the curvature of spacetime in the vicinity of the star is not what we expect.

Einstein's General Theory of Relativity is based on two principles. The first is that the geometry of spacetime is partly determined by the mass/energy which it contains and the second is that freely moving (inertial) objects follow geodesic paths through spacetime. Nobody would dare tamper with the second principle (which largely follows from the definition of an inertial object) but Einstein took 10 years to work out the details of the first principle and there is nothing in the theory that precludes there being causes of spacetime curvature other than the local presence of matter. Indeed, we have recently proved the existence of gravitational waves which are disturbances in the curvature of spacetime which propagate through space at the speed of light without the assistance of any nearby masses.

My proposal is this. In the absence of any nearby masses, *spacetime is not necessarily flat*. It may posses an *intrinsic curvature* which may vary from place to place and which has no external cause.

The rubber sheet analogy

The effect of a gravitating matter on the structure of spacetime is often illustrated using what is known as the rubber sheet analogy. A heavy spherical weight such as a cannon ball is placed in the centre of a circular rubber sheet stretched tight in a circular frame. The weight of the ball distorts the rubber into a funnel shape. A ball bearing thrown into the funnel orbits round the weight in a manner which is highly suggestive of the orbit of a planet round a star. The orbital period of a ball bearing travelling round the cannon ball will depend on the slope of the funnel at that radius. If the funnel has the shape of a rectangular hyperbola, the orbital period will obey Kepler's third law but if the funnel has a different shape (e.g. a logarithmic one) the speed of the ball bearing will be independent of the radius of the orbit.



The rubber sheet analogy

What I am suggesting is that if the fabric of spacetime is intrinsically curved in places, then the observed rotation curves can be explained without using dark matter. The problem of working out the distribution of dark matter needed to explain the observed rotation curve is replaced with the problem of working out what intrinsic curvature would produce the same effect.

The formation of galaxies

You may be wondering at this point why the proposed intrinsic curvature always appears to be spherically symmetric around the galaxy. What is to stop the curvature being greater on one side than the other? The answer lies in the way the galaxy was formed in the first place. The traditional theory of galactic formation is that the early universe (say, 100 million years after the Big Bang) consisted of a swirling mass of gas and dust, more or less uniformly distributed throughout space. Due to random fluctuations in density, there were some places where there was a slight accumulation of matter and, due to a positive feedback effect, these regions would attract more and more matter towards them. Owing to the law of conservation of angular momentum, the gas and dust would not simply fall to the centre but would start to rotate around a central core. Meanwhile, within this whirlpool of gas and dust, smaller concentrations of matter would form which would clump together, heat up and become stars.

This scenario is extremely plausible and support for the idea of initial fluctuations in density is found in the measurements of the Cosmic Microwave Background Radiation. But there is one problem. This process is relatively slow, especially in the early stages, and traditional models of galactic formation suggested that it would take 2 or 3 billion years for the first fully formed galaxies to form. Recently, however, deep field images of space taken first by Hubble and now by the James Webb Space Telescope have revealed far more galaxies in the early universe than expected, some little more than 500 million years old. What caused galactic evolution to proceed so rapidly?

Current theories of cosmic evolution use the concept of dark matter to help explain how the large scale structures of the universe evolved and how galaxies formed so quickly. But if dark matter does not exist, what alternative explanation is there?

Let us stretch (!) our rubber sheet analogy a bit further. Let us liken the early universe to a vast rubber sheet on which are scattered millions of marbles (representing particles of baryonic matter) in random motion. According to traditional theories, if there is a slight increase in density of the marbles in one region, there will be a slight lowering of the sheet in this region which will cause all the marbles nearby to migrate towards the lowest point. Eventually, so the theory goes, there will be a deep depression in the sheet with marbles swirling round it like stars in a galaxy.

Dark matter enthusiasts claim that in addition to the marbles the rubber sheet must be populated with five times as many tiny ball bearings which do not interact with the marbles and do not clump together in the same way. These ball bearings swiftly gravitate together to form 'filaments' or 'sheets' which attract the baryonic matter and precipitate the rapid formation of visible galaxies.

I am not competent to judge the success of these theories but my suggestion is that the ball bearings do not exist but that the rubber sheet (the stage on which the evolution of the early universe played out) was not flat but had slight mounds and depressions corresponding to primordial distortions of spacetime. Galaxies did not form at random anywhere, they formed where there were already primordial dimples. This explains nicely why the distortions in spacetime are always centred on the galactic core. A galaxy is not going to form on the 'side' of a 'dimple'. What is more, these imperfections in spacetime still exist and are the cause of the anomalous rotation curves of many galaxies. Lastly, since the process of galxy formation is 'seeded' by pre-existing variations in the structure of spacetime, we can expect the process to proceed much more rapidly than the traditional model predicts.

Gravitational lensing is just another example of spacetime distortion and is just as likely to be caused by 'dimples in spacetime' as it is to be caused by dark matter.

The CMB power spectrum

I have noted that the ΛCDM model can be constructed so that it generates a remarkably (suspiciously?) good fit to the observational data. I have already noted that without dark matter it is thought that the first peak would be much more prominent and the harmonics much less pronounced. How can these facts be reconciled with the idea that dark matter is replaced by 'dimples' in spacetime?

In the ΛCDM model, dark matter is considered to have a gravitational influence but no other significant property. It is essentially pure gravity. Assuming that it is uniformly distributed the net effect of all the dark matter in the early universe will be to give the universe an overall uniform positive curvature. Since dark matter has no other properties, it might as well be replaced with such a curvature. There is, however, one major difference. As the universe expands, the density of dark matter decreases according to an inverse cube law. Spacetime curvature, however, does not behave like this. To see why we need to go back to the Friedman equation which includes the term describing a curved universe. This equation is:

$$H^2 = \frac{1}{3}\Lambda + \frac{8\pi G}{3}\rho - k\frac{c^2}{a^2} \tag{9}$$

k is generally taken to have the values +1 for a closed universe, 0 for a flat universe and –1 for an open universe. We can do this because the scale factor a is arbitrary. Here, I wish to fix a such that a = 1 at the present time. This means that k can take any value and it represents the degree of primordial curvature.

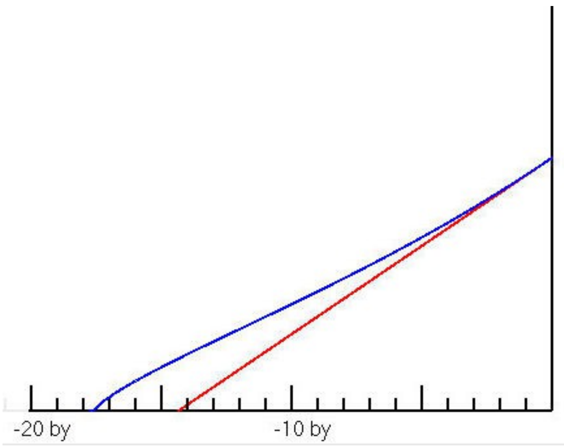
As we did before we can simplify this equation by putting all three terms into the same format . This is very simple. All we need do is add another term to equation (5):

$$\frac{da}{dt} = aH_0\sqrt{\Omega_\Lambda + \frac{\Omega_{m0}}{a^3} + \frac{\Omega_k}{a^2}} \tag{10}$$

where Ω_k is a constant.

As before, when a = 1 the expression under the square root sign must be equal to 1 so $\Omega_\Lambda + \Omega_{m0} + \Omega_k = 1$ If dark matter does not exists then Ω_{m0} = 0.05. The relative weights of the other two terms would have to be calculated to make the theory produce the same predictions as regards the CMB power spectrum. I am pretty confident that this can be done for the same reason that I think that anomalous galactic rotation curves and gravitational lensing can be explained without dark matter: WIMPS basically do nothing except create curvature so whatever a WIMP can do, primordial curvature can do equally well.

There is, however, one obvious difference between matter and curvature. The former scales according to an inverse cube law while the latter decreases according to an inverse square law. This will make the history of the universe's expansion look rather different. To take one example, if we keep Ω_Λ=0.68 then Ω_k=0.27. Under these circumstances it turns out that the universe must be 17.4 billion years old. The following graph shows the history of the universe according to the curvature model with these parameters:



Since the curvature term is positive, the value of k in the Friedman equation must be negative implying that the universe is open and therefore infinite in extent. The increase in the age of the universe from 13.8 by to 17.4 by would be good news for many astronomers.

The 'Flatness' problem

Another argument for the apparent flatness of the current unniverse is this. If the early universe was not exactly flat then the slight departures from flatness would have increased enormously since that time (some people say exponentially and a figure of 10^{60} is often mentioned) and the current universe would now be massively curved. If this was the case, it is argued, distant galaxies would appear massively magnified or diminished. Since this is not the case, the early universe must have been incredibly flat. It is much more likely (so the argument goes) that the early universe was exactly flat and that it has been flat ever since.

Lets look at this argument from the point of view of the Friedman equation.

Equation (9) can be written as:

$$H^2 = \left(\frac{da/dt}{a}\right)^2 = H_0^2(\Omega_\Lambda + \frac{\Omega_{m0}}{a^3} + \frac{\Omega_k}{a^2}) \tag{11}$$

where Ω_Λ. Ω_{m0}. and Ω_k are all constant. Since when $a = 1$, $H = H_0$, $\Omega_\Lambda + \Omega_{m0} + \Omega_k = 1$.

I shall now define a quantity C which I shall call 'curvature' such that

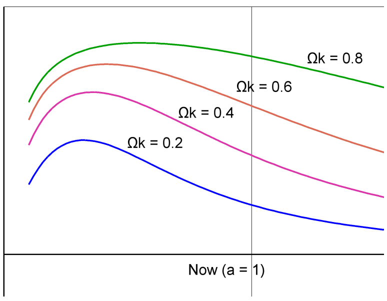
$$C = \frac{H_0^2 \Omega_k / a^2}{H^2} \tag{12}$$

This represents the proportion of H^2 which is contributed by the curvature term and which evaluates to $C = \Omega_k$ at the present time. Whether this quantity corresponds exactly to the meaning of the word 'curvature' as applied to 4 dime4nsional space time, I do not know. But it does not really matter. When this term is positive it means that there is insufficient matter and dark energy to make the universe flat and the universe is open; when it is negative there is an excess of matter and energy so the universe is closed.

Substituting for H we get:

$$C = \frac{\Omega_k / a^2}{\Omega_\Lambda + \Omega_{m0}/a^3 + \Omega_k/a^2} = \frac{\Omega_k}{\Omega_\Lambda a^2 + \Omega_{m0}/a + \Omega_k} \tag{13}$$

It is worth pausing a minute to see what this equation means. In an almost flat universe (Ω_k is small) which has no matter (i.e. the exponential universe referred to earlier) C decreases as the universe expands according to an inverse square law. Any departures from initial flatness are 'ironed out' by the expansion. In an almost flat universe containing *only* matter, C increases in proportion to a (not exponentially as some authors would have it[2]). Whatever the values of the constants, if the cosmological constant in not zero eventually the Ω_Λ term will dominate and the initial curvature will be ironed out. The following graph shows the behaviour of a universe with $\Omega_{m0} = 0.05$ and Ω_k varying from 0.2 to 0.8, the rest being dark energy (Ω_Λ).



In all cases, the curvature increases to a maximum and then starts to decrease. It definitely does not increase exponentially.

However, the flatness problem has not gone away entirely. It is still the case that in the early stages of the universe, the curvature would increase in proportion to a but the problem is no worse in a universe without dark matter than it is in a a universe with it.

2 The figure of 10^{60} which I mentioned earlier is not due to any 'exponential' increase, it is simply the amount by which the universe has supposedly expanded since the Planck era. Why we should take this as our starting point is anybodies guess. Why not start at zero?

If we are correct in assuming that a value $C = \Omega_k = 0.27$ will produce the same power spectrum at that predicted by the ΛCDM model, a positive value of Ω_k means that k in the Friedman equation (equation 9) is negative implying that the universe is open rather than closed.

If we assume, for the sake of argument, that 1 billion years after the Big Bang the universe is populated with a uniform distribution of essentially unchanging galaxies all the same size, then in a flat universe we would expect to see 4 times as many galaxies at a distance of $2r$ than at r. But in a curved universe, this relation no longer holds. In a closed universe the surface area of a sphere of radius r is smaller than $4\pi r^2$ and in an open universe, it is greater than $4\pi r^2$. When we look at very distant galaxies therefore, if the universe is closed we will see fewer galaxies than we would expect (and they would look larger too) whereas in an open universe we would see more (smaller) galaxies than expected. Recent observations by the James Webb Space Telescope have detected more billion year old galaxies than was expected. Is this evidence that the universe is, in fact open, not flat?

Criticism of the 'dimple' theory

Many will argue that the 'no dark matter' theory has just as many tweakable parameters as the ΛCDM model and is therefore no advance but it does make two predictions that can, in principle, be tested rigorously. It predicts a faster rate of growth of galaxies in the early universe and a longer timescale over which the present shape of the universe has formed.

The most serious flaw in the theory is that it gives no clue as to the origin of the dimples which are supposed to account for the anomalous rotation curves of the galaxies or the degree of observed gravitational lensing. Nor does it offer any explanation of why the universe should be open rather than flat. But then the origin of dark matter and even the universe itself is a mystery.

Of course, if it proves impossible to match the power spectrum of the CMB to the observational data satisfactorily then the theory will fall at the first fence. But falsifiability is the first requirement of a good theory. I do believe that the idea is worth pursuing.

Carr Bank: September 2026